Topological infrastructure analysis of the built environment

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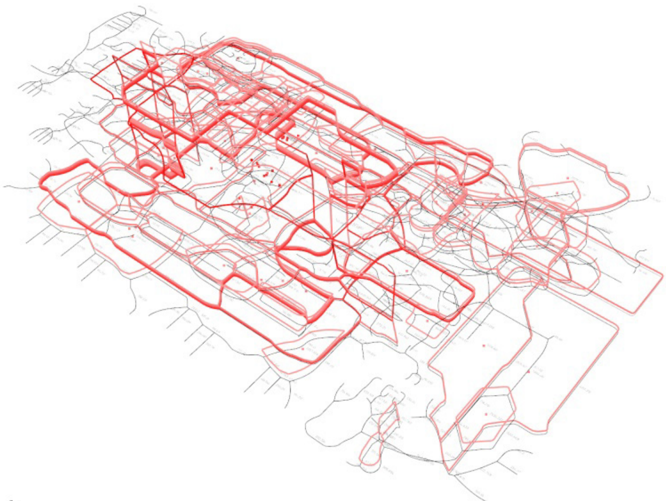
Abstract A key property of the built environment is the inherent circulation network describing how spaces connect. Conventional architectural representations are inadequate for representing and analysing physical infrastructure beyond material abstraction. A need for additional modes of representation which capture cognitive and organisational properties of space, such as circulation networks, thus exists. The scope of work described here is the development of formalized modes of architectural representation for mapping, abstracting and analysing these. Extending previous implementations of graphs for spatial analysis - axial maps, access and visibility graphs (Hillier, 1996), place graphs (Franz, 2008) and perceptual structures (Van Tonder, 2004) - the Medial Axis (MA) is proposed as a promising graph representation of topological infrastructure. A suite of dynamic CAD tools implementing algorithms from graph theory which lend themselves particularly well to spatial analysis has been developed. This enables the interactive generation of MA graphs based on floor plans, analysing these and immediately informing design based on the generated data. The tools have been tested for analysing spatial resilience related to security and safety concerns in building design and layout planning. This has yielded positive results and certain graph measures have proven to be particularly significant, such as node centrality, node degree and graph cycles.

Fig. 1 Identifying graph cycles in the complex infrastructure of the Banco De Londres bank in San Nicolás, Argentina



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Motivation

Conventional modes of architectural representation focus on the geometric description of physical infrastructure. While these are invaluable for capturing, analysing and communicating metric properties such as dimension, scale and shape, they do typically not manage to capture function or usage properties such as how environments are perceived or occupied by their inhabitants. These are thus insufficient for representing the built environment beyond a material level of abstraction. This suggests the need for additional modes of representation which more aptly capture cognitive and organisational properties of space significant to human spatial experience, behaviour and occupancy. We propose that this may be explored by shifting the subject of representation from quantitative to qualitative properties of space. That is, to focus on descriptive and implicit spatial properties as opposed to explicitly measurable properties.

A defining qualitative property in the layout of the built environment is the inherent circulation network describing how spaces are connected. Such networks not only represent spatial hierarchy but are equally instrumental in understanding building performance with regards to occupation and wayfinding behaviour. The scope of work described in this paper is the development of formalized modes of architectural representation for mapping, abstracting and analysing such networks.

Topological Infrastructure Analysis

A network is fundamentally a topological structure. That is, they describe qualitative connectedness for a set of components. In architectural terms how spaces connect: *topological infrastructure.* To effectively represent and analyse networks a mode of representation is necessary. In mathematics a graph is an abstract construct consisting of objects (nodes) where some pairs of these are connected by links (edges). These are highly flexible and extendable data structures which offer many algorithms for solving higher level queries which lend themselves well to spatial analysis. Graphs have previously been explored in an architectural context by means of axial maps, access and visibility graphs (Hillier, 1996), place graphs (Franz, 2008) and perceptual structures (Van Tonder, 2004). We extend this by exploring a synergy between Bill Hillier's topological classifications of spatial types represented through connectivity graphs and van Tonder's perceptual structures for spatial elements on a visual ground represented through medial axis graphs. Hillier's connectivity graphs generate topological hierarchies for spatial partitions, represented by nodes, which aim at understanding cultural configurations of space. Van Tonder uses medial axis graphs to demonstrate visual completion of spatially placed elements via Gestalt theoretical principles like continuity, enclosure and symmetry (Koffka, 1935). Both representations correlate spatial configuration with choices for movement.

Hillier discusses two graphs resulting from the geometry of space: the spatial layout as an adjacency complex (a-complex) and a movement structure as permeability complex (p-complex), of which multiple can be inscribed in an a-complex. He discovered that four types of nodes in p-complexes occur representing generic functions of spaces, mainly 'occupation' and 'movement': a-spaces as end-spaces for occupation; b-spaces as link spaces between occupation spaces; c-spaces as single-ring spaces and d-spaces as multiple-ring spaces. Rings represent cycles in circulation. B- and c-spaces are easier to control as all movement goes directly through them. A- and d-spaces are less controllable as they are either internally autonomous like a-spaces or are only one choice of route (redundancy of circulation) on multiple rings like d-spaces. Shallow configurations consist of a-spaces directly linked to c- or d-spaces, minimizing global spatial depth. Those represent smaller buildings of less complexity. Larger buildings create sub-complexes that are linked by controlled b- and c-spaces producing deeper topological depth and more complex way-finding.

Van Tonder's perceptual structures demonstrate how spatial elements in a visual field are cognitively structured at spatial locations where branching graphs converge. This convergence represents choices for movement towards adjacent spaces and facilitates global legibility. He therefore provides the perceptual user-centric interpretation at nodes through MA that Hillier classifies by global topological graphs. This paper illustrates the correlation between MA graphs and Hillier's spatial types of topological p-complexes to arrive at some interpretation for spatial resilience.

The Medial Axis

The *Medial Axis (MA)* of a given shape is a thin representation of this shape which is equidistant to the boundaries of the shape. It may be defined as a zero-area geometric model (a network of curves or points) which sits centrally in the given shape and forms a tree-like skeleton mapping the topology of the shape. It is consequently commonly known as the topological skeleton and is frequently applied in shape analysis for purposes such as character recognition, road network detection in GIS systems and meshing for finite element analysis in structural engineering. The MA graph network is isomorphic with the shape that generated it, that is, it possible to reconstruct the initial shape from the medial axis and vice versa.

Based on these properties the MA may be valuable as a model of representation for analysing the built environment which *"unites geometry and topology of the objects and the field they occupy in one single framework"* (Leymarie et al., 2008). Or, expressed in architectural terminology, the MA is capable of objectively capturing organisational and cognitive information of a given building, or urban plan, in the form of a graph that includes: the inherent route network, distances between walls, width of doors and passages, adjacencies of areas etc. The MA thus captures the inherent link between physical (shape) and spatial (topological) infrastructure and therefore demonstrates promise as the base graph for the development goals described in the previous sections.

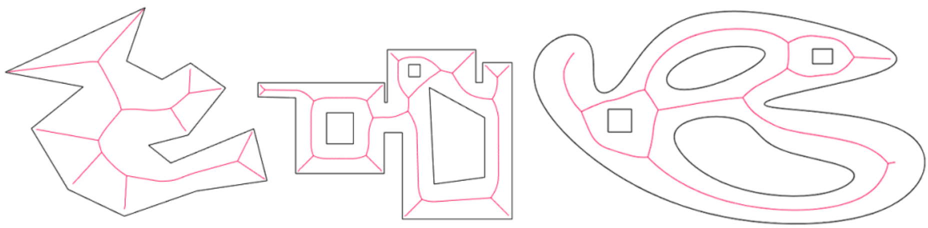

Fig. 2 The medial axis (magenta) extracted from various shapes and layouts (black) using our implementation

Graph Construction Methodology

Though standalone applications capable of generating the MA do exist, one of the primary goals with our research is to enable designers and architects the ability to analyse spatial layouts on the fly within already established workflows using existing drafting and representational software. Consequently, we have developed a library of dynamic CAD tools which allows the user to interactively generate MA graphs, analyse these and immediately inform design based on the generated data. These tools are developed for the Rhino 3D platform as custom Grasshopper components written in the Python programming language. The tools are wrapped into three main components: *MedialAxis2D*, *ReduceCurveGraph* and *CurveGraphAnalysis*.

The MedialAxis2D component extracts the MA from a two-dimensional representation of a spatial layout consisting of curves or polylines which are divided into a single *DomainBoundary* and a list of *DomainVoids*. While numerous methods for calculating the MA exist, our current method implements a resolution dependant approximation algorithm. In brief, the domain curves are subdivided by a given length and a Voronoi diagram is generated using these points of division. By removing any line segment in the Voronoi diagram which has an endpoint outside the DomainBoundary or inside any of the DomainVoids a raw MA graph is generated. By subsequently identifying points in which the remaining Voronoi line-segments connect in a node with a valence >= 3, we can construct a MA suitable for graph analysis by joining the line-segments into polylines which connect at these branching points.

The MA is inherently sensitive to the geometry of the domain curves. If these are overly detailed the MA calculation may become computationally expensive and result in superfluous information. Drafting the schematic representation of the building layout should therefore not exceed a level of detailing finer than needed. Depending on the type of analysis being carried out, the domain detail resolution should typically be roughly the size of person. However following this heuristic may still result in a MA in which graph edges are very short, and conversely, nodes which are very close. From the point of view of analysing spatial infrastructure such detailing adds little spatial information.

For example, if graph nodes represent *"points at which people need to make a decision regarding paths and/or directions to take"* (Tzeng & Huang, 2009), having several nodes within the radius of the human diameter, or the boundary description of what we would consider "a room", adds unnecessary information to the graph. Extending this, the outer edges of a MA graph extracted from a set of polylines will typically terminate in the concave control points of the input polylines. This enables us to identify corners, but as humans would often not consider walking into a corner an attractive option when moving about the built environment, this does not contribute to understanding the route network.

The ReduceCurveGraph component is designed to deal with such issues by reducing and refining the MA as a spatially meaningful sparse graph. A set of user defined controls define how the graph is reduced and rebuilt. These include: setting the level of which to recursively "prune" the outer branches of the graph, the minimum allowable edge length and whether or not through-nodes are allowed (nodes which only connect two edges). Additionally the component has controls for explicitly manipulating the graph by collapsing edges which are within a specified zone, adding nodes and removing edges, as well as rebuilding the output geometry to type and resolution. Fig. 2 demonstrates using the component to achieve a 71% graph edge reduction, while still maintaining a sufficient level of spatial and topological information to represent the spatial layout.

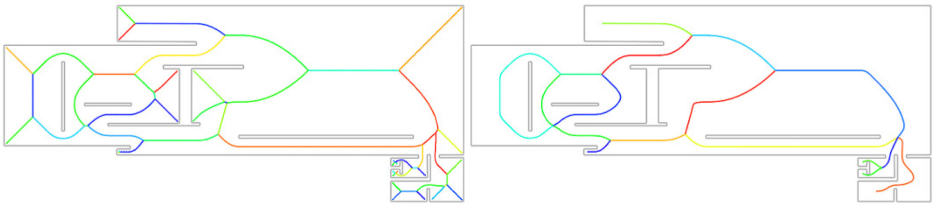

Fig. 3 Left: The Medial Axis (Edges: 59, Nodes: 56), Right: The reduced Medial Axis graph (Edges: 17, Nodes: 14)

Graph Analysis Methodology

In real-life scenarios the environment of spatial analysis will often exceed one floor. In such cases we employ a 2.5D stacking method. By extracting individual graphs for each floor and connecting these with graph representations of staircases and lifts, a continuous multi-layered 3D graph is generated which is capable of describing a building (or a city!) as one topological representation. An environment may furthermore not be directly representable in the 2D plane, for instance in cases with a high degree of topographic variation. In such cases we employ a 2.5D projecting method in which the domain curves are projected to the plane. Subsequently the MA graph is generated and projected back to the space of the initial geometry.

Once a graph has been generated and reduced the CurveGraphAnalysis component may be employed to evaluate the graph by applying measures and methods from graph theory. In this mode of representation the built environment is discretized by means of nodes and edges extracted from the MA graph, corresponding to sub-spaces or locations within the plan (nodes) and how these connect spatially (edges). This provides a model in which nodes and edges carry local information about their immediate context as well as global information about their position in the overall graph network. The current version of the CurveGraphAnalysis component offers multiple modes of analysis and visualisation for nodes, edges and graph cycles as well as the option to export the generated data to an Excel file for further statistical analysis.

Using the component it is possible to identify properties and characteristics of the spatial layout such as dead ends, through points, branching points and cycles in the topology graph. Through combining the graph with the initial domain representation it is furthermore possible to extract properties such as the relative size and visibility of the nodes. By implementing measures from graph theory to globally evaluate the nodes it is furthermore possible to assign values to nodes such as their integration and centrality in the graph network as well as their distance to exits or certain functions within the environment. Global evaluation may also be useful for calculating classic graph measures such as shortest paths etc.

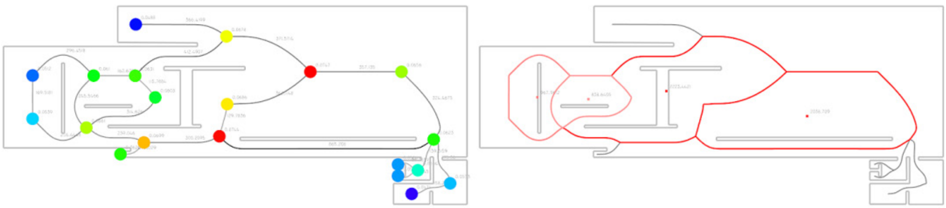

Fig. 4 Running graph analysis on the reduced MA. Left: Closeness Centrality, nodes which are centrally located in the graph (reds: high values, blues: low values). Right: Graph Cycles, edges which form a closed circuit

Applications to Spatial Resilience

Resilient Infrastructure and Building Security (RIBS) is an EU funded Framework 7 research project which addresses the protection of building infrastructure and how this relates to building design and engineering. We are conducting the infrastructure and spatial analysis for the project, developing a suite of methods for evaluating spatial resilience, visibility, occupancy patterns, and organisational structure and proximity. In this context the presented graph tools have been applied for the analysis of existing buildings with regards to resilience and risk assessment. Specifically they have been implemented to run spatial analysis on a high-risk building in Europe simply referred to as *"The Object"*. For dissemination purposes and to verify the applicability of the tools we have furthermore ran analysis on selected decommissioned banks: *the National Farmer's Bank of Owatonna* (Minnesota, USA, Louis Sullivan, 1908), *Laenssparbanken Falun* (Falun, Sweden, Hultman & Holmer, 1973) and the *Headquarters of the Banco de Londres y America del Sur* (San Nicolás, Argentina, Clorindo Testa, 1959). This process has yielded positive results and certain graph measures have proven to be particularly significant to spatial analysis and resilience alike:

Graph Cycles

The term cycle refers to a closed path within a graph i.e. locations which connect back to themselves via a walkable route, correlating to Hillier's c- and d-types. As a measure of spatial resilience graph cycles expose circulation properties within the building layout and identify specific locations which are particularly vulnerable when exposed to a threat. If a location is not part of a cycle it may lack spatial redundancy with regards to number of possible routes in egress and escape scenarios, correlating to Hillier's a- and b-types. Conversely a node which is part of two or more cycles may be considered to have a high level of redundancy. If a node with this property is occupied, or taken out by attackers, it will not affect the global accessibility of the network on the same level as a node which is only part of a single cycle (c-type) or not part of one at all. The property of being part of several cycles (d-type) may also be interpreted as a measure of likelihood of being travelled upon. A location with a high amount of traffic is likely the more attractive target in certain threat scenarios. Conversely a central location which is not on a cycle may also be attractive, as taking it out will effectively split the circulation network.

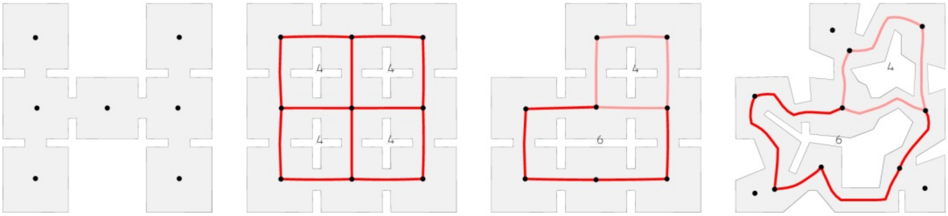

Fig. 5 Graph cycles identified on a range of spatial configurations. Red saturation indicate amount of constituent nodes. Note that this colour gradient is graph dependant

Node Degree

The degree, or valence, of a graph node is defined as the number of edges which connect in this particular node i.e. how many adjacent locations are spatially connected to a given location. In network analysis the node degree is often interpreted as a measure of the immediate risk of a node being exposed to an entity passing through the network (such as a computer virus). When applied to spatial resilience this interpretation can be seen as a measure of how exposed a location or space is to a physical threat passing through the building i.e. the more connections a location has, the higher its chance of exposure to a potential risk is. Node degree is furthermore valuable for identifying significant building locations with regards to navigation and choice, such as dead ends (a-type) which have a node degree of one, and branching points (c- and d-type) which have a node degree higher or equal to three.

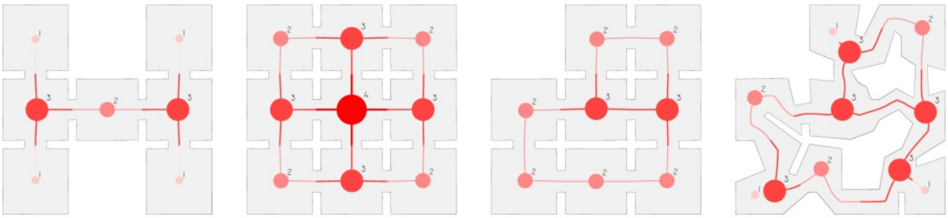

Fig. 6 Node degree visualised as sized dots, red saturation indicating amount of connecting edges

Node Centrality

Where node degree may be seen as the local centrality of a node, a global centrality measure is defined by what is referred to in graph theory as *"closeness centrality"*. In a connected graph a distance between any two nodes in the graph may be determined by the length of the shortest path between these two. By calculating the sum of shortest path lengths from a node to all other nodes in the graph the *"farness"* of the node is determined. The closeness centrality of the node is defined as the inverse of this measure. A node is thus considered more central the lower its total distance to all other nodes. In network analysis closeness centrality is regarded as a measure of how long it will take to spread data from a node to all other nodes in the graph. Hence it may be interpreted as a measure of how well a location is integrated in the overall physical network of a building. When applied to spatial analysis and resilience, node centrality is consequently an indicative measure for evaluating how isolated or exposed a location is in the route network of a building. This may also be seen as a measure of node importance in terms of navigation and thus probability of threat exposure (in the case of intruders). Node centrality may also be calculated for the purpose of evaluating network integration with regards to specific locations (such as emergency exits) or objects (such as important assets). The CurveGraphAnalysis component is furthermore capable of calculating the related node measure *"betweenness centrality"*.

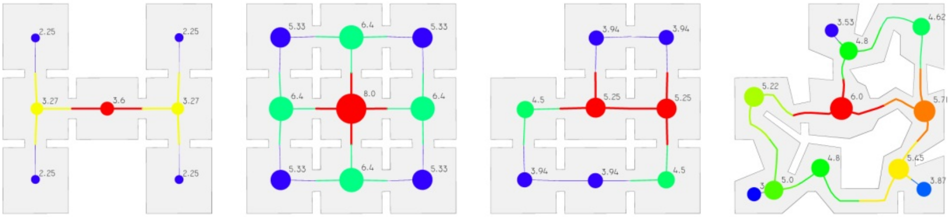

Fig. 7 Closeness centrality visualised as hues on a gradient ranging from low (blue) to high (red)

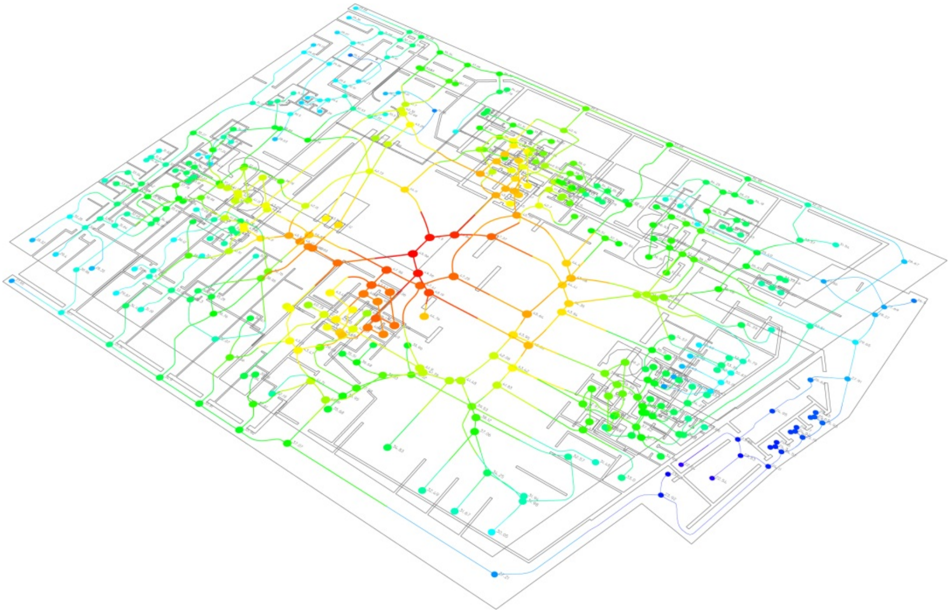

Fig. 8 Closeness centrality calculated on the reduced MA graph of the two-story Laenssparbanken Falun Bank

Conclusion

We have demonstrated the potential of the Medial Axis and graph theory measures for analysing the topological infrastructure of the built environment. Novel features include the dynamic generation of the MA within a CAD environment, the algorithms for reducing and refining the MA into a spatially meaningful sparse graph, the implementation of measures and methods from graph theory for dynamic analysis and evaluation of the graph, the application of the work in the context of a highly specific field of spatial analysis in the RIBS project.

As demonstrated by Derix and Miranda (Leymarie et al., 2008), MA graphs can furthermore be applied to urban structures. Apart from providing an alternative automatically generated axial graph for mapping permeability complexes at an urban scale, the MA inherently represent the shape of spatial areas through equidistant axes. For urban planning, the notion of appropriation of space is key for defining not only defensible spaces but also creating legible implicit ownership of spaces, leading to safe and well-used territories. For squares this is particularly interesting, as a MA would show the partitions into territories of appropriation, creating gradients of "publicness".

Gradients would have to be represented by the values of the distance field between the generating edges and the equidistant axes, i.e. not the graph itself.

Clearly, the MA as discussed here can be applied for urban movement structures and transferring van Tonder's perceptual structures would help to understand way-finding choices in more depth, particularly if measures such as node degree and angular deviation were introduced to other standard way-finding criteria like longest edge or number of turns (Dalton, 2001). Hillier and Steadman's notion of "minimizing depth" as clear way-finding structures can be enabled through research on cycles and angular deviation, giving clear mental images of a route and urban structure as proposed by Kevin Lynch (Lynch, 1960).

Future work thus includes exploring the MA graph applicability for large scale urban analysis and the mapping of perceptual properties of space onto node and edge types. This includes identifying further potentially significant graph properties and measures. For this purpose a faster MA algorithm, capable of real-time interaction on large and complex models, will likely need to be implemented. Finally it is anticipated that the assumptions made regarding the MA graph for anticipating circulation and occupancy is further tested by performing a one-to-one real life case study, comparing surveillance data from a building with its derived MA graph measures.

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